

# Compact Homodyne Extrapolation System (CHEXS)

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**Abstract**—We present on a novel gain extrapolation antenna range, the **C**ompact **H**omodyne **E**xtrapolation **S**ystem (CHEXS), that can achieve absolute antenna gain measurements with uncertainties of +/-0.1 dB or better with as few as 10 data points and is significantly more compact, up to six times shorter than conventional gain extrapolation ranges. This compact gain extrapolation range achieves these beneficial attributes by measuring the homodyne signal that occurs naturally between two directional antennas that often exhibit strong third order mutual coupling at close proximity. The design and operation of the CHEXS is presented along with gain measurements of NIST reference standard gain antennas which are shown to be equivalent to those obtained using a conventional gain extrapolation range.

## I. INTRODUCTION

Conventional gain extrapolation (a.k.a. *extrap*) antenna ranges are optimized to implement the well-established gain extrapolation technique [1]-[4]. The gain extrapolation technique which has been in use for over 50 years is considered one of the most accurate techniques for measuring antenna gain routinely achieving uncertainties of less than +/- 0.1 dB. One of the popularly known advantages of using gain extrapolation is that the perturbations to the gain [5] due to antenna-to-antenna multiple-reflections, proximity effects, and environmental multi-path effects can be observed and removed from the measured data using filtering techniques to isolate the direct antenna coupled signal. Many techniques have been reported on ways to filter gain extrapolation data to remove these unwanted effects [6]-[10]. As such, conventional gain extrapolation antenna ranges are designed around the intention that the *direct signal* is most important and should be the focus of the measurement. This notion is a driver for gain extrapolation facility design which includes aspects such as chamber size, automation architecture, data acquisition, and post processing.

In this paper we describe a novel type of gain extrapolation antenna range the **C**ompact **H**omodyne **E**xtrapolation **S**ystem (CHEXS) that uses the Enhanced Gain Extrapolation Technique [10] and is designed around the intention of measuring the homodyne signal (as opposed to only isolating the direct signal)

that occurs naturally with directional antennas that often exhibit strong mutual coupling under the gain extrapolation scheme. As we will discuss below, the benefits of the CHEXS over conventional gain extrapolation ranges are: 1) only 10 data points need to be acquired as opposed to the thousands of data points needed with conventional gain extrapolation ranges and 2) the maximum antenna-to-antenna distance can be up to six times shorter than in conventional gain extrapolation ranges.

## II. CHEXS ARCHITECTURE DESCRIPTION

### A. Setup

As in traditional gain extrapolation a means of boresight alignment and ability to translate antennas along the boresight axis in order to dynamically change the separation distance,  $d$ , is required. This is accomplished by mounting the antennas on multi-axis positioners such as robotic arms or stacked motion stages [3],[4],[11],[12]. Depending on the range of motion of the positioners a linear rail may need to be added to achieve the required farthest antenna separation distance.

### B. Measurements and Data Sampling

Fundamental to the operation of the CHEXS is its implementation of the enhanced gain extrapolation technique [10]. The enhanced technique provides a means to extract the pair gain of two coupled antennas from the homodyne signal without the need to filter the data, rather than trying to isolate the direct signal through filtering techniques as is done in the conventional gain extrapolation technique. In the enhanced gain extrapolation technique it is shown that the homodyne signal,  $I$ , that results from the interference of the direct wave and the *third order* scattered wave as a function of the antenna separation distance  $d$ , is given by,

$$I(d) = I_0(d) + I_1(d) + 2\sqrt{I_0(d)I_1(d)} \cos\left(\frac{4\pi d}{\lambda} + \Delta\varphi\right), \quad (1)$$

where,

$$I(d) = (|S_{ij}|d)^2. \quad (2)$$

Where  $S_{ij}$  is the off-diagonal scattering coefficient (S-parameters) between the ports of the two antennas that are being scanned over the separation distance,  $d$ . For example, if using a two-port calibrated vector network analyzer (VNA), with each antenna connected to each port, the signal would be  $S_{12}$  or  $S_{21}$ .

As the antenna separation is varied the observed homodyne signal consists of a spectrum of interference fringes of varying amplitude with maximum and minimum extrema spaced a distance of  $\lambda/4$  apart. Often referred to as “multiple reflections”, here we aim to be more precise and concise in considering these features as they predominantly result from interference of the third-order wave with the direct wave and provide insights into the wave propagation characteristics of the antennas. This homodyne signal is dominant for directional antennas such as standard gain horn (SGH) antennas where the third order scattered wave is significant. As opposed to being a nuisance, as is the case in conventional gain extrapolation ranges, this signal in fact can be advantageously used to extract the pair gain of the two antennas and thus the individual antenna gains. The use of the homodyne signal also has the added benefits of allowing significantly less data points to be used and shorter distances. An example of a set of these interference fringes that obtained with a CHEXS at 12.4 GHz for a pair of NIST reference X-band antennas [14] is shown in Fig. 1.

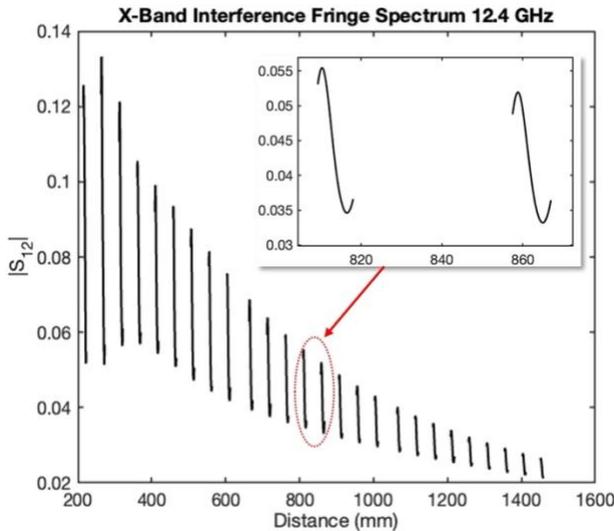


Figure 1. Example of interference fringes obtained with a CHEXS at 12.4 GHz for X-band standard gain horn antennas that result from direct-wave and third order scatter waves. Inset shows a close up of the fringes centered at 813 mm and 861 mm.

To obtain the gains from the homodyne signal (1), the pair of extrema of each interference fringe (maximum,  $I_{max}$ , and minimum,  $I_{min}$ ) centered on the antenna-to-antenna separation distance,  $d$ , are measured and averaged to directly give the antenna coupling coefficients as such,

$$\frac{I_{max} + I_{min}}{2} = \sum_{n=0}^4 \frac{A'_{0n}}{d^n}. \quad (2)$$

These coefficients, up to the fifth term  $A'_{04}$  and including the desired gain coefficient  $A'_{00}$ , may be extracted by solving equation (2) using the extrema,  $I_{max}, I_{min}$ , for five sets of interference fringes occurring at five distances,  $d$ . A minimum of 10 measurements are needed (five fringes with two extrema per fringe). More fringes can be measured if desired however as shown below results are stable for 10 or more measurements. It can be shown [3],[4] the gain coefficient,  $A'_{00} = (g_i g_j)/(4k^2)$ , where  $g_i$  and  $g_j$  are the gains (in linear units) of each antenna “i” and “j” in the pair. The product of the gains or pair gain,  $g_i g_j$ , for three sets of antennas  $g_1 g_2, g_2 g_3$ , and  $g_1 g_3$  are determined in this way and used to calculate their individual gains using the well-known three antenna method [3]. As only 10 measurements are needed, the CHEXS produces significantly less data than the thousands of measurements typically acquired using a conventional gain extrapolation range [3].

### C. Antenna Separation Distance Considerations

By targeting the homodyne signal, the CHEXS can be made shorter than a conventional gain extrapolation range. The third order wave, which drives the homodyne signal, experiences three times the travel distance compared to the direct wave for a given antenna separation distance,  $d$ , which in effect compresses the length requirements. In particular, when considering the length needed for the CHEXS the farthest antenna separation distance,  $d_{Far}$ , can be shorter by a factor of three compared to a conventional gain extrapolation when the Rayleigh distance,  $d_R = 2D^2/\lambda$  is used and by a factor of six when  $d_R = 4D^2/\lambda$  is used as the far distance in conventional gain extrapolation. Where,  $D$  is the largest antenna aperture dimension and  $\lambda$  is the operating wavelength. The range of near and far distances used in the CHEXS as given by the enhanced gain extrapolation method [10] are thus,

$$d_{Near} = 0.62 \sqrt{\frac{D^3}{\lambda}}, \quad (3)$$

and,

$$d_{Far} = \frac{2D^2}{3\lambda}, \quad (4)$$

respectively. The CHEXS acquires the 10 needed measurements (discussed in Section II A.) spread out between these near and far distances. Figure 2. shows a comparison of the dimensions of a CHEXS vs a traditional gain extrapolation range. For comparison, for standard gain horn antennas, in a conventional gain extrap range,  $d_{Far}$  can reach up to 9 meters where as the CHEXS would require only 1.5 meters. Table I. shows a comparison of  $d_{Far}$  between the CHEXS and the conventional gain extrapolation for the NIST reference antennas. The range of values  $[2D^2/\lambda, 4D^2/\lambda]$  for the Rayleigh ranges used in the conventional extrapolation range are given in the  $d_{Far}$  column.

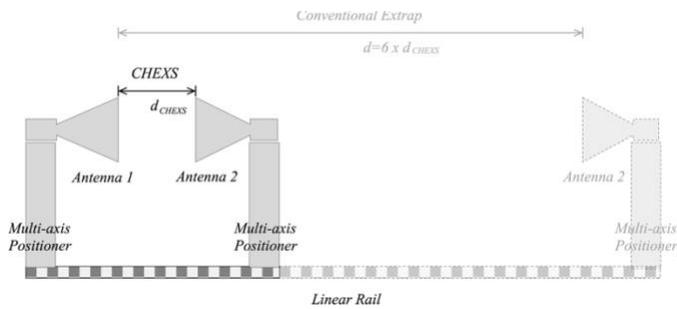


Figure 2. Length comparison (to-scale) of CHEXS vs a conventional gain extrapolation range. The antenna separation,  $d$ , for the conventional extrapolation range is up to six times greater than for the CHEXS.

TABLE I. FAR DISTANCE COMPARISON

GHz	D (m)	CHEXS		Conventional
		$d_{Near}$ (m)	$d_{Far}$ (m)	$d_{Far}$ (m)
8	0.25	0.40	1.11	[3.33, 6.67]
10	0.25	0.45	1.39	[4.17, 8.33]
12	0.25	0.49	1.67	[5, 10]
12.4	0.20	0.36	1.10	[3.31, 6.61]
15	0.20	0.39	1.33	[4, 8]
18	0.20	0.43	1.60	[4.8, 9.6]

#### D. Data Acquisition Architecture

Since only a select number of extrema (max and min) of the homodyne interference fringes are needed the sampling strategy of the CHEXS can be more strategic and efficient than those used in conventional gain extrapolation ranges. As a result, the automation architecture is based on coordinating the acquisition of the RF signal with the targeted positions of the max and min of extrema each of the five fringes. This is fundamentally different than in conventional gain extrapolation ranges, where the automation architecture is based on a continuous capture of data points at a regular spacing of typically  $\lambda/10$  or  $\lambda/20$  for the purposes of filtering out the interference fringes rather than measuring them specifically. Moreover, these continuous finely spaced sampling results in large data sets typically thousands of data points per antenna pair.

The sampling scheme for CHEXS can be broken down into two main steps. In the first step the location of the first maximum (or minimum) is found and its value acquired. Next, subsequent extrema are located based on their calculable periodicity based on the homodyne signal (1). As shown in [10] the locations of the interference fringe extrema of order,  $l$ , is given by,

$$d_{max} = l \left( \frac{\lambda}{2} \right), \quad (7)$$

$$d_{min} = \frac{(2l + 1)\lambda}{4}. \quad (8)$$

For the first step, the antenna positioner is first initialized to the position corresponding to antenna separation,  $d_{Near}$  followed by scanning the antenna separation distance while monitoring

the value of the transmitted signal i.e. be  $S_{12}$  or  $S_{21}$ . Signal monitoring to locate the first extrema may be done by monitoring the transmitted signal analog output in real-time or by acquiring a small sample of data points of a fraction of a wavelength and interpolating or differentiating on the fly to find the maximum or minimum. The position and value of  $S_{21}$  of the first extrema is stored and the positions of the remaining four sets of extrema are calculated using (7) and (8). The antenna positioner is then commanded to move to the subsequent four extrema and the values of  $S_{21}$  along with the value of,  $d$  measured.

#### E. Measurement Examples

Measurements of four NIST reference antennas were used to validate the CHEXS concept. Measurements were taken varying the number of data points from the minimum of 10 up to 15 to demonstrate the stability of the results. The standard deviation (not standard deviation of the mean) for the set of gain values obtained from the 10 to 15 measurements was included as an uncertainty term in the final expanded uncertainty. The uncertainty analysis is shown in Table II. A comparison of the gain values obtained with the CHEXS to the reference values are given in Figure 3 (below at end of paper). The Y-axis of each plot in Figure 3 shows the difference in dB between the gain values measured with the CHEXS and the reference values for the antennas. The X-axis is the number of measurements used for each Y-value and ranges from 10 to 15 measurements. The length of the error bars on each gain value data point are those from Table II. The horizontal dotted lines in each plot are the upper and lower uncertainty bounds for the respective reference values. These bounds are equal for antennas 1 and 2 (“Ant1” and “Ant2”) for all frequencies except 15 GHz where there are then two sets (blue and red) of upper and lower bound dotted lines. From Figure 3 it is seen that for all frequencies and for all number of measurements (10 through 15) the gain values obtained with the CHEXS fall within the reference gain values and associated uncertainty bounds (dotted lines).

#### F. Conclusion

In this paper we present on a novel gain extrapolation range that intentionally uses the homodyne signal produced by antenna-to-antenna mutual coupling. The CHEXS is up to six times shorter than a conventional gain extrapolation range and only needs to take 10 data points to determine absolute antenna gain. An overview of the CHEXS is presented. Measurement results of four NIST reference antennas at 8 GHz, 10 GHz, 12 GHz, 12.4 GHz, 15 GHz and 18 GHz obtained with the CHEXS are presented and shown to be within the stated gain values of the reference antennas with uncertainties of +/- 0.1 dB or better.

TABLE II. UNCERTAINTY ANALYSIS

Antenna 1						
	8 GHz	10 GHz	12 GHz	12.4 GHz	15 GHz	18 GHz
RF Stability (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Impedance Mismatch (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Alignment (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Connector Repeatability (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Multipath (dB)	0.02	0.02	0.02	0.02	0.02	0.02
Random (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Gain Standard Deviation (dB)	0.01	0.02	0.01	0.04	0.02	0.01
<b>U, Expanded Uncertainty (k=2) (dB)</b>	<b>0.06</b>	<b>0.07</b>	<b>0.06</b>	<b>0.1</b>	<b>0.07</b>	<b>0.06</b>
Antenna 2						
	8 GHz	10 GHz	12 GHz	12.4 GHz	15 GHz	18 GHz
RF Stability (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Impedance Mismatch (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Alignment (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Connector Repeatability (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Multipath (dB)	0.02	0.02	0.02	0.02	0.02	0.02
Random (dB)	0.01	0.01	0.01	0.01	0.01	0.01
Gain Standard Deviation (dB)	0.02	0.02	0.01	0.01	0.01	0.03
<b>U, Expanded Uncertainty (k=2) (dB)</b>	<b>0.07</b>	<b>0.07</b>	<b>0.06</b>	<b>0.06</b>	<b>0.06</b>	<b>0.09</b>

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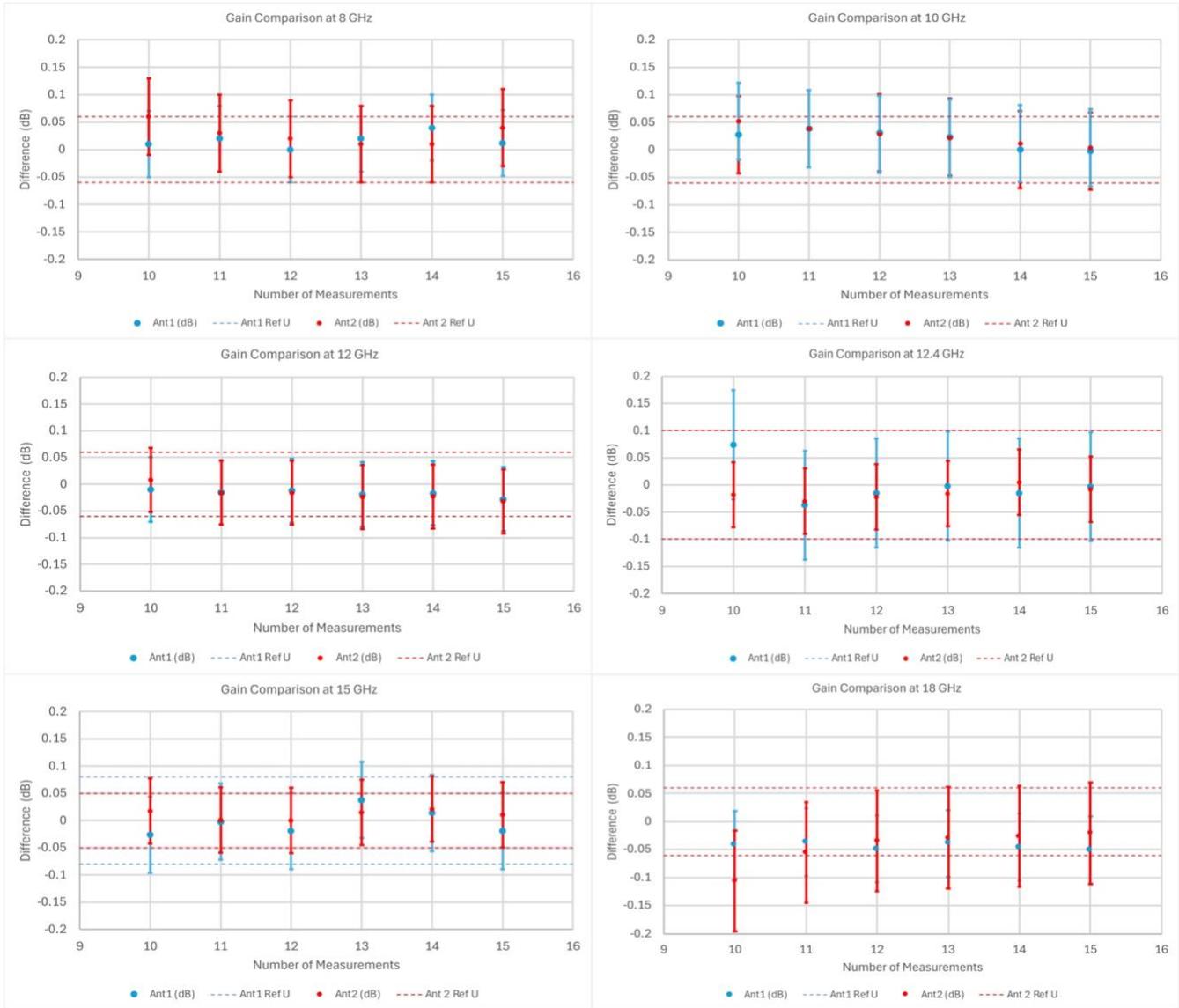


Figure 3. For each frequency, shown in each plot are the difference between the reference gain values and the measured gain from the CHEXS as a function of the number of data points used. Antennas 1 and 2 are denoted as Ant1 and Ant2 respectively in the legends. The uncertainty (taken from Table II) for each gain value is given by the error bars. Dotted lines for upper and lower bounds for the reference gain uncertainty (taken from [13] and [14]) for antennas 1 and 2 are denoted in the legends as “Ant1 Ref U” in blue and “Ant2 Ref U” in red respectively. These bounds are equal for Ant1 and Ant2 for all frequencies, except 15 GHz, where there are then two sets (blue and red) of upper and lower bound dotted lines.